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PLANAR MOTION CONTROL, COORDINATON AND DYNAMIC ENTRAINMENT IN CHAPLYGIN BEANIES

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ABSTRACT

The Chaplygin beanie is a single-input robotic vehicle for which partial planar motion control can be achieved by exploiting a simple nonholonomic constraint. A previous paper suggested a strategy for such motion control. In the present paper, this strategy is validated experimentally and extended to the context of multi-vehicle coordination. It is then shown that when the plane on which two such vehicles operate is translationally compliant, energy transfer between the two can enable a mechanism whereby one (operating under control) may entrain the other (operating passively), partly coordinating their motion. As an extension to this result, it is further demonstrated that a pair of passive vehicles operating on a translationally compliant platform can eventually attain the same heading when released from their deformed configurations.

INTRODUCTION

Control laws that mimic passive physical phenomena are employed in diverse mechanical settings. An actuator that applies a force proportional to the deviation in a mechanical element's measured position from a desired value, for instance, can be considered to act as a linear spring. Mechanical analogies in turn motivate a variety of control designs. Successful strategies for the coordination of multi-vehicle systems, for instance, have been developed whereby individual vehicles respond to movement relative to their neighbors as if linked to these neighbors by linear springs and dampers [1]. Real physical coupling can also complement deliberate control in the context of coordination. Systems of aerial or aquatic vehicles that seek states of motion mimicking bird flocks or fish schools, for instance, may do so using feedback control but may benefit from the local attractiveness of states that represent local energetic minima [2, 3]. The tendency for physically coupled nonlinear systems to synchronize may be so strong as to overcome contrary regulation at the individual level — a point observed as early as Huygens' experiments with synchronizing pendulum clocks in the 1660s [4].

The system depicted in Fig. 1 was introduced in [5] and named the *Chaplygin beanie* because it combines elements of two canonical systems from the mechanics literature: the *Chaplygin* (or *Carathéodory*) *sleigh* (essentially a modern shopping cart, supported in the front by casters and in the rear by one or more wheels that roll without slipping [6]) and *Elroy's beanie* (a rigid body coupled through an actuator to a balanced rotor surmounting its center of mass [7]). It was demonstrated analytically in [5] that this system will translate from rest if the rotor is induced to spin relative to the cart, and that a simple proportional control law mimicking the action of a torsional spring on the rotor can be used to dictate the cart's asymptotic heading and longitudinal speed simultaneously. An analogy was also developed between this system and a planar fishlike swimmer, each able to propel itself by exploiting a resistance to lateral motion at its rear through periodic variations in angular momentum forward of this resistance. A more direct analogy obtains between the Chaplygin beanie and the aquatic vehicle studied subsequently in [8], consisting of a rigid planar hydrofoil with an internal balanced rotor. In section 1 of the present paper, a physical realization of the system in Fig. 1 is presented and used to validate the control

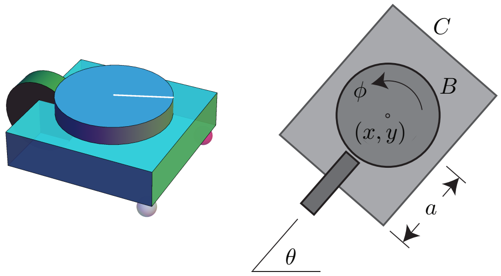

FIGURE 1. The Chaplygin beanie. The position of the cart's center of the mass and the orientation of the cart relative to a stationary frame of reference are specified by (x, y) and θ, respectively. The rotational inertias of the rotor and cart relative to their point of coupling at the cart's center of mass are denoted by B and C, respectively. The mass of the system overall is denoted by m in the text. The rotational inertia of the rear wheel, which can roll freely but cannot slip laterally, is considered to be negligible.

approach suggested in [5]. In section 2, a strategy is outlined to generalize this approach to the problem of coordinating the orientation and translational speed of two or more Chaplygin beanies. In section 3, numerical results are presented that document a surprising phenomenon whereby a passive Chaplygin beanie with a torsional spring in place of an actuator will reorient itself and follow a self-propelling Chaplygin beanie when the two are supported by a common platform through which vibrational energy can be transmitted. Entrainment of this kind is patently analogous to the synchronization of pendulum clocks sharing vibrational energy through a common mantel, but is also arguably analogous to a form of entrainment that occurs within schools of fish that exchange kinetic energy through wake vorticity. A dramatic illustration of the latter is documented in [9]: the flexible body of a dead fish can be induced to swim against a background current toward a bluff object upstream when excited by vorticity shed periodically from the object. Finally, in section 4, numerical results are presented demonstrating entrainment in a pair of passive Chaplygin beanies atop a translationally compliant platform. The entrainment documented in section 3 and 4 is interesting in its own right as a new example of unexpected behavior engendered by nonholonomic constraints — the spin reversal of the rattleback is a classic, contrasting example [6] — but may also suggest a future strategy for coordinating the motion of systems of underactuated vehicles like the Chaplygin beanie through decentralized control. F

1 SINGLE-VEHICLE DYNAMICS AND CONTROL

1.1 Analysis

The dynamics of the system shown in Fig. 1, with a torque inducing the rotor to rotate relative to the cart serving as the solitary control input, are those of a control-affine system with drift. The configuration manifold $\mathbb{S}^1 \times \mathrm{SE}(2)$ is four dimensional, but the constraint

$$-\dot{x}\sin\theta + \dot{y}\cos\theta = a\dot{\theta}$$

prohibiting lateral slipping of the rear wheel on the ground decreases the dimension of the system's phase space by one. The machinery of nonholonomic reduction [10] was employed in [5] to isolate the dynamics of the two-dimensional nonholonomic momentum with scalar components

$$\begin{aligned} J_{LT} &= m\dot{x}\cos\theta + m\dot{y}\sin\theta, \\ J_{RW} &= -m\dot{x}a\sin\theta + m\dot{y}a\cos\theta + (B+C)\dot{\theta} + B\dot{\phi} \end{aligned}$$

representing, respectively, the system's forward translational momentum and its angular momentum relative to a vertical axis passing through the center of the rear wheel. The system's drift dynamics are given overall by

$$\begin{aligned} \dot{x} &= \frac{J_{LT}}{m}\cos\theta - a\left(\frac{J_{RW} - B\alpha}{ma^2 + B + C}\right)\sin\theta \\ \dot{y} &= \frac{J_{LT}}{m}\sin\theta + a\left(\frac{J_{RW} - B\alpha}{ma^2 + B + C}\right)\cos\theta \\ \dot{\theta} &= \frac{J_{RW} - B\alpha}{ma^2 + B + C} \\ \dot{J}_{LT} &= ma\left(\frac{J_{RW} - B\alpha}{ma^2 + B + C}\right)^2 \\ \dot{J}_{RW} &= -a\left(\frac{J_{RW} - B\alpha}{ma^2 + B + C}\right)J_{LT} \\ \dot{\phi} &= \alpha. \end{aligned} \tag{1}$$

Normalizing with respect to the rotational inertia of the rotor, we can consider the rotor's angular acceleration $\dot{\alpha}$ to be under direct control.

A fundamental obstacle to local controllability is present in (1). The forward momentum J_{LT} is nondecreasing, and any change in the cart's heading is necessarily accompanied by an increase in this momentum.[1] Nevertheless, if the system is initially at rest, then it's possible to induce the cart to accelerate using a simple proportional controller so that its heading and translational speed — to wit, θ and J_{LT} — will approach any desired values asymptotically. Without loss of generality, the desired asymptotic value of θ can be taken to be zero. It's shown in [5] (essentially via LaSalle's invariance principle) that the feedback

[1] With appropriate magnitude, a sinusoidal torque on the rotor will induce the system to accelerate from rest in an undulatory fashion. A movie of this is visible at http://tinyurl.com/gm4h2nh.

law $\dot{\alpha} = k\theta$ will drive θ to zero for any positive value of the gain k. The closed-loop system will evolve, furthermore, so that the energy-like quantity

$$\Lambda = \frac{(J_{RW} - B\alpha)^2}{ma^2 + B + C} + \frac{J_{LT}^2}{m} + kB\theta^2$$

is conserved over time. It follows that

$$\lim_{t\to\infty} J_{LT}^2 = m\Lambda(0) = D + kF, \qquad (2)$$

where

$$D = m\frac{(J_{RW}(0) - B\alpha(0))^2}{ma^2 + B + C} = m(ma^2 + B + C)\dot{\theta}(0)^2$$

and

$$F = mB(\theta(0))^2.$$

If $\theta \neq 0$ and $\dot{\theta} = 0$ initially, then k can be selected to assign any positive value to $\lim_{t\to\infty} J_{LT}$. If $\theta \neq 0$ but $\dot{\theta} \neq 0$ initially, then k can be selected to assign any positive value to $\lim_{t\to\infty} J_{LT}$ that's greater than a certain lower bound.

1.2 Experimental results

Fig. 2 depicts a physical realization of the Chaplygin beanie. This system's dynamics deviate from those of the model (1) principally because of friction associated with the rear wheel, resisting both pivoting of the cart (due to friction between the wheel and the ground) and forward rolling (due to friction in the bearings between the cart and the wheel). We demonstrate experimentally that if the system in Fig. 2 is initially at rest and its rotor is driven thereafter according to a feedback law akin to $\dot{\alpha} = k\theta$, then the heading angle θ will approach zero over time and the cart's forward speed will approach a value that increases with increasing k. The controller we implement is merely "akin to" the proportional controller from section 1.1 in two practical ways. First, the actuator coupling the rotor to the cart is a small brushed DC gearmotor[2] powered by a pulse-width modulated (PWM) input voltage. The feedback gain k multiplies the cart's heading to determine not the motor torque directly but the duty cycle of the PWM signal. Second, limitations on the speed and torque available from the motor confine our experiments to a regime in which friction plays a visible role. One way to overcome the cart's tendency to lose forward momentum while stabilizing its heading is to increase its tendency to overshoot this heading, since oscillations in heading generate a propulsive force according to the third and fourth lines in (1). We therefore augment the proportional feedback of section 1.1 with an integral feedback term to increase overshoot, scaling the integral gain with the proportional gain from one experiment to the next. It's expected that future versions of the physical apparatus in Fig. 2 will permit experiments involving more aggressive maneuvers that downplay the short-term influence of friction, obviating the need for integral feedback. On the other hand, it's expected that additional experiments with the current apparatus will contribute new understanding to the phenomenon of *damping-induced heading recovery*, a manifestation of damping-induced self-recovery [11] doc-

FIGURE 2. A physical Chaplygin beanie constructed primarily from one inch–thick extruded polystyrene foam. In place of casters, the aluminum pie plates hover on cushions of air generated by counter-rotating ducted fans to support the forward end of the cart with minimal friction between the cart and the ground. Batteries contribute significantly to the inertia of the cart; the rotor's large diameter helps to increase its rotational inertia without increasing the system's mass overall.

[2]`https://www.pololu.com/product/2386`

umented in the context of dissipative aquatic locomotion in [12].

Figure 3 depicts the results of three different experiments with the system from Fig. 2. In each experiment, feedback linking the angular acceleration of the rotor to the cart's heading induces the cart to execute a right-angle turn. The translational speed attained by the cart in doing so increases with increasing gain.

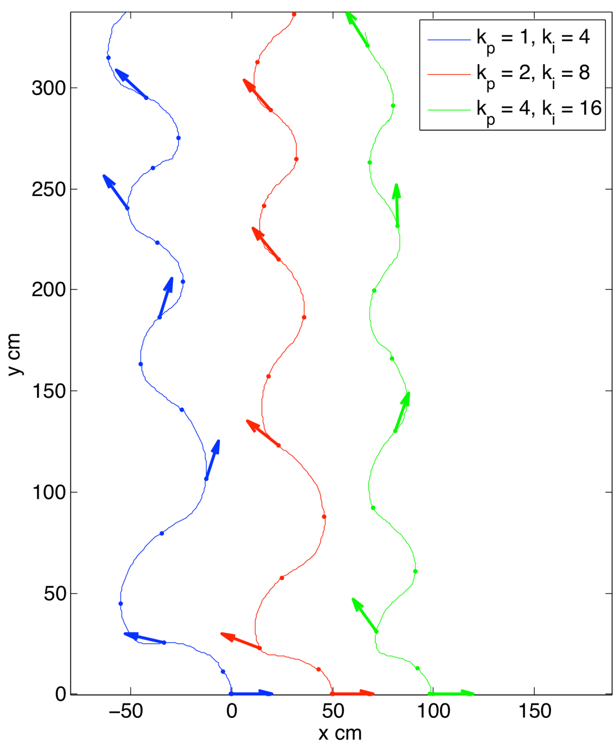

FIGURE 3. Trajectories of the physical Chaplygin beanie under the influence of three different heading controllers. In each case, the cart is initially centered at rest along the *x* axis and pointing in the positive *x* direction. A PI controller is activated to stabilize the heading to the positive *y* direction, resulting in forward locomotion as well as rotation. The dots along the trajectories represent equal intervals of time, showing that an increase in feedback gain results in an increase in the cart's translational speed. The desired average heading is eventually achieved in every case.

2 HEADING AND SPEED COORDINATION

Imagine two identical Chaplygin beanies at rest, situated so that their initial headings differ. If proportional heading control is applied to each to steer it toward the initial average heading of the two, and if the same gain k is used in both feedback loops, then the two will approach a state asymptotically in which they translate in the same direction at the same speed. The quantity

$$\Lambda_{\text{decoupled}} = \frac{(J_{RW\,1} - B\alpha_1)^2 + (J_{RW\,2} - B\alpha_2)^2}{ma^2 + B + C} + \frac{J_{LT\,1}^2 + J_{LT\,2}^2}{m} + kB\left(\theta_1^2 + \theta_2^2\right)$$

will be conserved as their dynamics evolve, furthermore, even if their initial average heading doesn't correspond to $\theta = 0$.

An alternative strategy for coordinating the asymptotic heading and speed of the two vehicles is to implement the pair of control laws

$$\dot{\alpha}_1 = k(\theta_1 - \theta_2), \quad \dot{\alpha}_2 = k(\theta_2 - \theta_1) \tag{3}$$

so that each vehicle attempts to stabilize its heading to that of the other as the two headings evolve. A comparison of the two strategies appears in Fig. 4. Using the same shared gain k in both cases, the second strategy results in more rapid convergence to the system's final state. In the second case, the quantity

$$\Lambda_{\text{coupled}} = \frac{(J_{RW\,1} - B\alpha_1)^2 + (J_{RW\,2} - B\alpha_2)^2}{ma^2 + B + C} + \frac{J_{LT\,1}^2 + J_{LT\,2}^2}{m} + kB(\theta_1 - \theta_2)^2$$

is conserved. The quantities $\Lambda_{\text{decoupled}}$ and Λ_{coupled} differ in the "potential energy" terms that reflect the sense in which the control torques on the two rotors may be associated metaphorically with torsional springs.

Recall from (2) that the final translational speed of a solitary Chaplygin beanie subject to proportional heading control depends on both the feedback gain and the initial error in its heading. Because of this, any number of such vehicles with distinct initial headings can be induced to approach any common heading and translational speed asymptotically through separate control. Though the details aren't presented here, the authors propose that a set of feedback laws interconnecting the dynamics of a collection of such vehicles in a manner that generalizes (3) can always be constructed to achieve the same objective, potentially more efficiently, based on the shaping of an potential energy–like term involving a distribution of gains. This will be the topic of a future paper.

3 VIBRATIONAL ENTRAINMENT

Again imagine two identical Chaplygin beanies at rest, each situated and oriented arbitrarily. Suppose that the rotor atop one

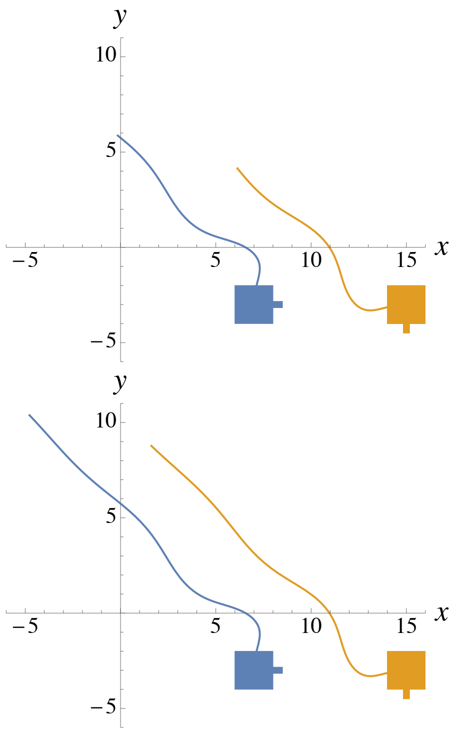

FIGURE 4. Trajectories of two Chaplygin beanies subject to proportional control stabilizing their headings to a common value. The top panel depicts the pair's response to the decoupled control laws $\dot{\alpha}_i = k(\theta_i - \theta_{\text{desired}})$, the bottom panel to the control laws (3). Both panels correspond to the interval $0 < t < 20$ and $m = a = B = C = k = 1$ for both vehicles.

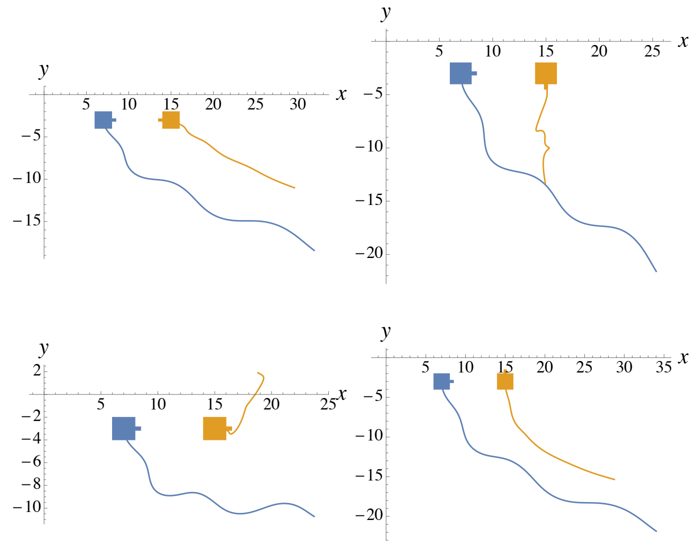

FIGURE 5. Initial trajectories leading to the entrainment of a spring-driven Chaplygin beanie (orange) by a sinusoidally actuated Chaplygin beanie (blue) on a shared platform that exhibits finite translational inertia. The trajectories shown are measured relative to the platform. The parameters m, a, B, and C are set to unity for both vehicles, as are the mass of the platform and the stiffness of the torsional spring coupling the orange rotor and cart. The blue rotor pivots relative to the blue cart so that $\phi = \sin t$. The two vehicles are shown in their initial configurations in each panel. Since the forward translational speed of each is necessarily nondecreasing, a trajectory that appears to indicate translation in reverse actually reflects reorientation followed by translation. Each panel represents the interval $0 < t < 20$.

cart is actuated but the rotor atop the other is coupled to the cart beneath through a linear torsional spring. Suppose, furthermore, that the two vehicles rest atop a common platform that isn't rigidly fixed but instead exhibits translational compliance with finite inertia. It's clear from the analysis of section 1.1 that if the rotor atop the first cart is induced to pivot sinusoidally, then the cart will advance relative to the platform, exerting a time-varying force on the platform that will induce the platform to move as well. It's also relatively clear that if the platform begins to vibrate, this will induce the second cart to pivot about its rear wheel, in turn exciting the second rotor to oscillate, stimulating forward propulsion.

What's unapparent is whether or not a relationship exists between the actuated vehicle's eventual direction of translation and that of the passive vehicle. At this point in the narrative, the system resembles an asymmetric variant of Huygens' clocks: two oscillators share a substrate through which vibrational energy can be transmitted, and as a result the oscillatory dynamics of one drive similar oscillatory dynamics in the other. The reader might anticipate that the passive vehicle will be attracted to a state of translation parallel or antiparallel to the translation of the actuated vehicle, observing that the rear wheel of the passive vehicle will be maximally responsive to lateral forcing and minimally responsive to longitudinal forcing, but this reasoning suggests no bias between parallel and antiparallel states.

In fact, the passive vehicle will eventually follow the actuated vehicle even if doing so requires a complete reversal in the former's heading. The asymptotic difference between the average heading of the one vehicle and the average heading of the other isn't zero — we compare *average* headings because both vehicles describe persistently undulatory paths — but remains small regardless of the initial position and orientation of the one relative to the other. This is illustrated in Figs. 5 and 6. Fig. 5 depicts initial trajectories corresponding to four distinct initial conditions. Some of these trajectories suggest a rapid convergence of the two vehicles' headings, others do not. Fig. 6 depicts the long-term average difference in the vehicles' headings as a function of the initial difference for two different initial relative positions. This long-term average difference is less than $\pi/20$ radians in almost every case, indicating a robust tendency

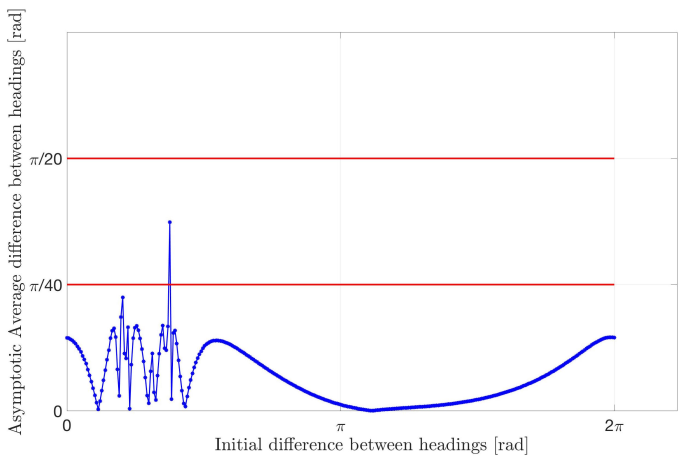

FIGURE 6. Asymptotic average relative orientation as a function of initial relative orientation for the pair of Chaplygin beanies (one active and one passive) as depicted in Fig. 5

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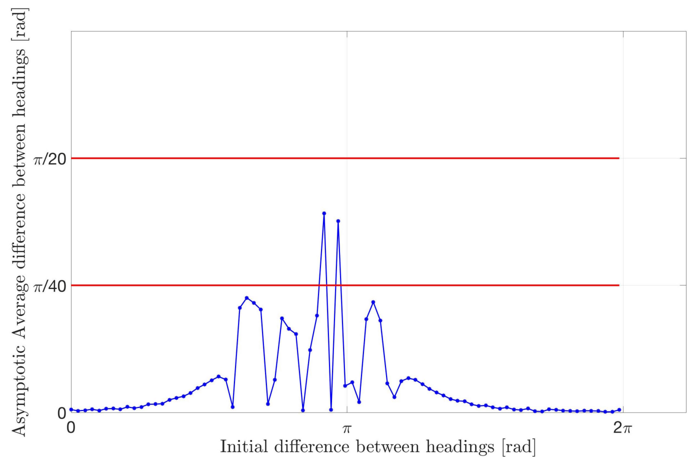

FIGURE 7. Asymptotic average relative orientation as a function of initial relative orientation for the pair of passive Chaplygin beanies atop a translationally compliant platform. For this simulation, the mass of the platform relative to mass of the beanie is $M_{platform}/m_{beanie} = 0.01$

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for the actuated vehicle to entrain the passive vehicle in a manner suggestive of the entrainment that can occur when one body propels itself past another in a fluid.

4 ENTRAINMENT IN PASSIVE BEANIES

As a further extension to the scenarios presented before, imagine now a situation with two passive beanies resting atop a translationally compliant platform, except that now both beanies are coupled to their respective rotors with linear torsional springs. In this system, there is no active controller commanding the movement of the rotor in either beanie. We would like to explore whether it is possible to induce synchronized motion in the two beanies by releasing them on the platform from a non-zero initial deformation in the torsional springs. As one can imagine, the potential energy stored in the deformed springs would leak into kinetic energy as the springs unwind, which would result in forward translation of individual beanies. We would like to explore the conditions under which the coupling between the two beanies is sufficiently strong so as to induce synchronized motion between them.

Much like the case where two beanies were both commanded by proportional heading controllers attempting to align the heading of each beanie towards that of the other's (Eq.(3)), one would expect here that depending on the initial difference in the headings and a parameter describing the coupling, both beanies will translate in completely parallel or antiparallel directions in the steady state. In the case of active control of two beanies, the coupling parameter was governed by the gain of the proportional control law. In this case, the relative magnitude of the mass of the platform and mass of a beanie govern the asymptotic difference between the average headings of the individual beanies. In Figure 7, we show the asymptotic difference between the average headings of the two passive beanies as a function of the initial difference. For this simulation, one of the beanie's initial heading was held fixed while the other beanie's orientation was swept through a 2π range. Once again, the asymptotic difference in the average heading is bounded above by $\frac{\pi}{20}$ *i.e.* $\lim_{t \longrightarrow \infty} |\theta_1(t) - \theta_2(t)| \leq \frac{\pi}{20}$. We are currently investigating an analytical formalism using a Lyapunov function to analytically prove that such a bound exists for this autonomous system.

Additionally, the mass of the supporting platform plays a role in the exchange of energies between the beanies. Greater inertia of the platform for a fixed beanie mass and rotor inertia will limit the synchronization. Indeed as can be seen in Figure 8, as the mass of the platform relative to the mass of the beanies is increased beyond a critical value of 0.155, the heading entrainment breaks, reminiscent of a bifurcation.

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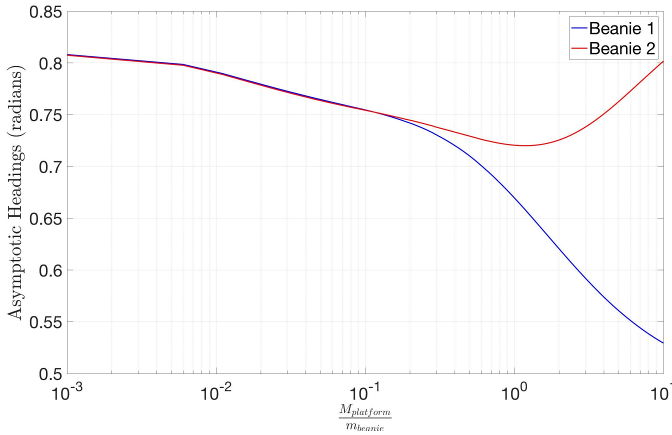

FIGURE 8. Effect of increasing the mass of the platform relative to the mass of individual beanies on the asymptotic headings. For this simulation, $\theta_1(0) = 0, \theta_2(0) = \frac{\pi}{5}$

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